PARABOLIC SYSTEMS WITH DYNAMIC BOUNDARY CONDITIONS: NULL CONTROLLABILITY AND INVERSE PROBLEMS

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Abstract. In this talk, we present our new results on null controllability and inverse problems of the parabolic equation with dynamic boundary conditions and drift terms

$$\begin{cases} \partial_t y - d\Delta y + B(x).\nabla y + c(x).y = 1_\omega u + f & \text{in } \Omega_T, \\ \partial_t y_\Gamma - \delta\Delta_\Gamma y_\Gamma + d\partial_\nu y + b(x).\nabla_\Gamma y_\Gamma + \ell(x) y_\Gamma = 1_{\Gamma_0} v + g & \text{on } \Gamma_T, \\ y_{|\Gamma}(t,x) = y_\Gamma(t,x) & \text{on } \Gamma_T, \\ y(0,\cdot) = y_0 & \text{in } \Omega, \\ y_{|\Gamma}(0,\cdot) = y_{0,\Gamma} & \text{on } \Gamma, \end{cases}$$

where Ω is a bounded domain of $\mathbb{R}^N$, with smooth boundary $\Gamma = \partial\Omega$ of class C^2, $\nu(x)$ is the outer unit normal field to Ω in the point $M(x)$ of Γ, $\partial_\nu y := (\nu.\nabla y)_{|\Gamma}$, d, δ are positive real numbers, $c \in L^\infty(\Omega)$, $\ell \in L^\infty(\Gamma)$, $B \in L^\infty(\Omega)^N$, $b \in L^\infty(\Gamma)^N$, $f \in L^2((0,T)\times\Omega)$ and $g \in L^2((0,T)\times\Gamma)$. The functions u and v are internal and boundary controls, acting on small regions ω and Γ_0, respectively. To obtain our aim, we show first some suitable Carleman estimates for the backward adjoint problems.